Exam "Toepassingen van de Quantum-Fysica" Groningen, 5-11-99

Please each problem on a separate sheet of paper (to facilitate correction by different people). Please put your name on each sheet!

Problem 1 *Hyperfinestructure of Na*

In order to capture Na atoms in a magneto-optical trap by means of laser light one has to tune the laser to the appropriate frequencies. The ground state of Na(3s)$^2S_{1/2}$ is split into hyperfine levels due to the interaction of the electron cloud with the nuclear magnetic moment μ_I. For the 'normal' isotope ^{23}Na, which has a nuclear spin of I=3/2, this moment has a magnitude of $\mu_I = 2.2161\ \mu_N$.

a) Into how many hyperfine levels is the ground state of ^{23}Na split?
b) give the characterization of these levels in terms of quantum numbers F for the 'normal' isotope ^{23}Na.
c) We now consider the isotope ^{24}Na, which has a nuclear spin I=4 and a nuclear g-factor of $g_I = 0.423$. Into how many hyperfine levels is the ground state split here, and what are the corresponding F quantum numbers.
d) We know that the hyperfine splitting in the ground state of ^{23}Na is 1772 MHz. How large is the corresponding splitting for the isotope ^{24}Na?

$$\Delta E_{HFS} = \frac{a}{2}[F(F+1) - I(I+1) - J(J+1)].....with......a = \frac{g_I \mu_N B_J}{\sqrt{J(J+1)}}$$

Problem 2 *Decelerating atoms with circularly polarized light in a magnetic field (Zeeman Slower)*

^{87}Rb (I=3/2) atoms with velocity v=500 m/s are evaporated from an oven and pass the inhomogeneous magnetic field of a Zeeman slower. Counterpropagating photons emitted from a laser (λ=780 nm) are absorbed and the atoms are slowed down stepwise. We consider transitions between m_F states of $5S_{1/2}$ and m_F states of $5P_{3/2}$ induced by a laser operating at a wavelength λ=780 nm. Consider transitions between the F=2 ground state (g_F=1/2) to the F=3 excited state (g_F=2/3) level.

a) Sketch the energetic positions of the magnetic substates outside and inside the Zeeman slower (B=0 and B weak, respectively).
b) Why has the field to be inhomogeneous to slow down the atoms?
c) Using σ^+ polarized light only transitions with Δm_F=+1 are induced. Which m_F levels are finally involved in the optical pumping and why?
d) Calculate the energy difference as a function of B.
e) How many absorption processes are necessary to slow the atoms down to 200 m/s assuming only spontaneous emission?
f) What is the length of the slower (the spontaneous emission lifetime of the 5p state is 26 ns)?
g) Calculate the B-field as a function of the position
h) How does the pumping scheme change when using σ^- polarized light (Δm_F=-1)? Discuss a possible advantage of using σ^- polarized light instead of σ^+ polarized light when slowing down atoms to very low velocities. Hint: What happens to atoms slowed down to v~0 when they leave the slower, i.e. the magnetic field goes to zero?

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Problem 3 — *Electron in a screened Coulomb potential*

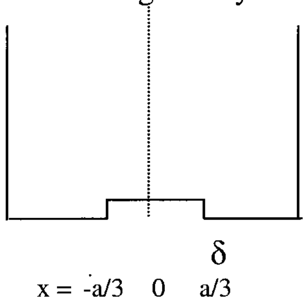
The quantum defects for the alkali atom lithium (Li, Z=3) are experimentally known: $\Delta n(s)=0.4115$, $\Delta n(p)=0.04$ and $\Delta n(d)= 0.001$.

a) Which photon energy is needed to excite the lithium from the groundstate Li(*2s*) to the states Li(*2p*) and Li(*3p*)?
b) If we describe the binding energy with an effective charge Z_{eff} instead of a quantum defect: what is Z_{eff} for Li(*2p*) and Li(*3p*)
c) Why is the effective charge different for Li(*2p*) and Li(*3p*)?
d) A photon removes a *1s* electron from excited Li ($1s^2 2p$). Determine the possible term symbols of the resulting ion.
e) What is the energetic order of the terms given in d)?
f) If in a second excitation step the configuration $Li^+(2p^2)$ is formed: what are now the possible terms and their energetic order?

Problem4 — *Perturbation*

Consider the one-dimensional square 'stepped' potential well given by

$V(x) = +\infty$ for $|x|>a$
$V(x) = 0$ for $a/3 < |x| < a$
$V(x) = \delta$ for $|x| < a/3$
where δ is small with respect to the energy of the lowest level



a) what are the lowest wavefunctions for the undisturbed potential ($\delta=0$)
b) what are the corresponding eigenenergies?
c) discuss, why first order perturbation theory is appropriate to determine energies and wavefunctions for the disturbed potential
d) from first order perturbation theory find the two lowest energy levels,
e) give the general expression for the corresponding new wavefunctions. Are some terms vanishing in these expressions?